

Stellar mass black holes in star clusters: gravitational wave emission and detection rates

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We investigate the dynamics of stellar-mass black holes (BH) in star clusters focusing on the dynamical formation of BH-BH binaries, which are very important sources of gravitational waves (GW). We examine the properties of these BH-BH binaries through direct N-body computations of Plummer clusters, having initially $N(0) \leq 10^5$ low mass stars and a population of stellar mass BHs, using the state-of-the-art N-body integrator “NBODY6”. We find that the stellar mass BHs segregate rapidly into the cluster core to form a central dense sub-cluster of BHs in which BH-BH binaries form via 3-body encounters. While most of the BH binaries finally escape from the cluster by recoils due to super-elastic encounters with the single BHs, we find that for clusters with $N(0) \gtrsim 5 \times 10^4$, typically a few of them dynamically harden to the extent that they can merge via GW emission within the cluster. Also for each of such clusters, there are a few escaped BH binaries that merge within a Hubble time, most of the merger happening within a few Gyr of cluster evolution. These results imply that the intermediate-aged massive clusters constitute the most important class of star clusters that can produce dynamical BH-BH mergers at the present epoch. The BH-BH merger rates obtained from our computations imply a significant detection rate ($\approx 30 \text{ yr}^{-1}$) for the “Advanced LIGO” GW detector that will become operative in the near future. Finally, we briefly discuss our ongoing development on this work incorporating the formation of BHs in star clusters from stellar evolution. In particular, we highlight on the effect of stellar metallicity on the BH sub-cluster driven expansion of a star cluster’s core.

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1. Introduction

Star clusters, *e.g.*, globular clusters (henceforth GC), young and intermediate-age massive clusters harbor a large overdensity of compact stellar remnants compared to that in the field by virtue of their higher stellar density and the mass segregation of the stellar remnants. These compact stars, which are neutron stars (hereafter NS) and black holes (hereafter BH), are produced through the evolution of the most massive stars in the cluster within the first ~ 50 Myr after cluster formation. Since the BHs are significantly heavier than the remaining low-mass stars of the cluster, they undergo a runaway mass-segregation or mass-stratification instability [1] provided a significant fraction of them are retained in the cluster following their progenitor supernovae. Due to this instability (also known as the Spitzer instability), the BHs concentrate quickly to the cluster core forming a dense sub-cluster made purely of BHs which is nearly dynamically isolated from rest of the cluster.

Earlier studies of the dynamical properties of such BH sub-clusters (*e.g.*, references [2–4]) indicate that the density of the BHs there is large enough that the BH-BH binary formation through 3-body encounters [5] becomes important. These dynamically formed BH binaries then shrink in orbit or “harden” through repeated super-elastic encounters with the surrounding BHs (Heggie’s law [6]). The binding energy of the BH binaries released in this way is carried by the single BHs and BH binaries involved in the encounters. This causes the single and the binary BHs to get ejected from the BH core to the outer regions of the cluster and they deposit their energy to the cluster stars while returning to the core via dynamical friction [1, 5], thereby heating the cluster. This heating mechanism is most efficient in the central region of the cluster since the stellar density is there the highest. As the BH binaries harden, the encounter-driven recoils in the BH core become increasingly stronger and finally the recoils are large enough that the encountering single BHs and/or the BH binaries escape from the cluster. This also results in heating of the cluster core as the associated mass-loss dilutes the potential well there. These heating mechanisms result in an expansion of the central part of the cluster and modify the dynamical evolution of the cluster significantly.

In the present work, we make a detailed study of the dynamics of the BH-BH binaries formed in a BH sub-cluster. Specifically, we investigate whether hard enough BH binaries, that can merge via gravitational radiation in a Hubble time within the cluster or after being ejected, can be formed in such a sub-cluster. To that end, we perform a series of direct N-body integrations of concentrated star clusters (half-mass radius $r_h \leq 1$ pc) consisting of $N \sim 10^4 - 10^5$ low-mass stars, in which a certain number of stellar-mass BHs are added, representing a star cluster with an evolved stellar population. However, we also consider initial clusters with a full stellar mass spectrum in which the BHs are formed from stellar evolution.

2. Computations

To study the dynamics of BHs in star clusters, we perform direct N-body integrations of model star clusters using the “NBODY6” N-body integration code running on Graphical Processing Unit (GPU) platforms [7]. The initial clusters follow a Plummer model with half-mass radius $r_h(0) \leq 1$ pc consisting of $N(0) \leq 10^5$ low-mass main-sequence stars in the mass-range $0.5M_\odot \leq m \leq 1.0M_\odot$.

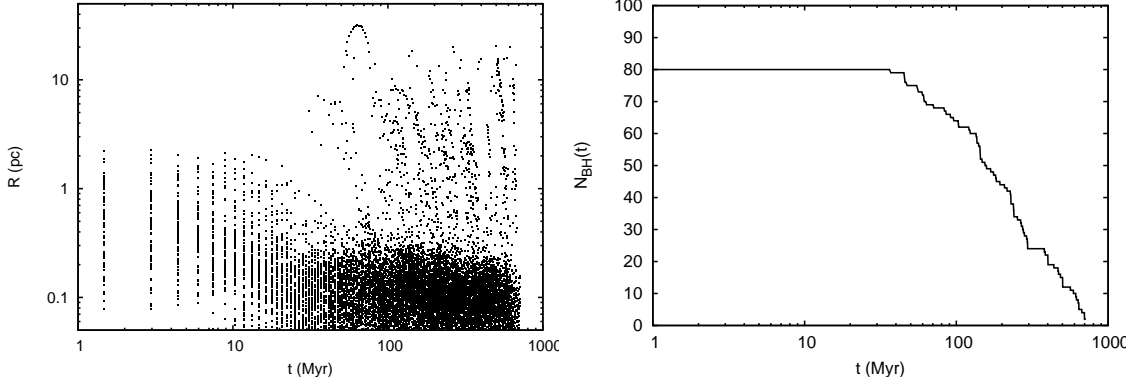


Figure 1: Mass segregation of BHs shown by the radial position R vs. time t (left panel) for the model C50K80. The BHs segregate in ~ 50 Myr during which N_{BH} (the number of BHs bound to the cluster) remains constant (right panel). As the BH sub-cluster becomes dense enough that BH-BH binaries begin to form via 3-body encounters, single BHs and BH binaries start escaping from the cluster depleting N_{BH} .

A specified number of BHs are added to each cluster with the same initial Plummer distribution as the stars. The initial number of BHs are chosen according to the BH retention fraction, where we explore both the full retention and the case where half of the BHs have escaped from the cluster by supernova natal kicks. For simplicity, we consider only equal-mass BHs in the present work, with the representative value of $M_{BH} = 10M_{\odot}$. Such an initial condition mimics the epoch at which the massive stars have already been evolved to produce their remnant BHs.

To evolve the BH-BH binaries due to GW radiation, the well known Peters' formula is adapted in NBODY6, according to which the merger time T_{mrg} of an equal-mass BH-BH binary due to gravitational wave (GW) emission is given by,

$$T_{mrg} = 150\text{Myr} \left(\frac{M_{\odot}}{M_{BH}} \right)^3 \left(\frac{a}{R_{\odot}} \right)^4 (1 - e^2)^{7/2}, \quad (2.1)$$

where, a is the semi-major-axis of the binary and e is its eccentricity. Studies in numerical relativity indicate that for unequal-mass BH-BH mergers or even for equal-mass BHs with unequal spins, the merged BH product acquires a velocity-kick of typically 100 km s^{-1} or more due to the asymmetric momentum outflow from the system, associated with the asymmetry of the GW emission. Therefore, in our computations, we provide an arbitrarily large velocity kick of 150 km s^{-1} to the merged BH immediately after a BH-BH merger occurs within the cluster, to make sure that it escapes. Also, we evolve the clusters isolated, *i.e.*, in absence of a galactic tidal field, as the formation of the BH-core through mass segregation and its dynamics remain largely unaffected by the presence of a tidal field, which mainly influences the stars near the tidal boundary. Furthermore, for simplicity, we do not consider primordial binaries in this initial study. See reference [4] for the details on the implementation of initial conditions and GW natal kicks.

2.1 Mimicking a GC core: reflective boundary

We also perform computations with a smaller number of stars and BHs that are confined within a reflecting spherical boundary [4]. With such a dynamical system, one can mimic the core of a

Table 1: Summary of the computations for isolated clusters and those with reflective boundary. The different columns represent as follows: Col. (1): identity of the particular model — similar values with different names (ending with A,B etc) imply runs repeated with different random seeds, Col. (2): total number of stars $N(0)$ initially, Col. (3): number of computations N_{sim} with the particular cluster, Col. (4): initial half-mass radius of the cluster $r_h(0)$ (isolated cluster) or radius of reflective sphere R_s , Col. (5): initial number of BHs $N_{BH}(0)$, Col. (6): total number of BH-BH binary mergers within the cluster N_{mrg} , Col. (7): the times t_{mrg} corresponding to the mergers, Col. (8): number of escaped BH-pairs N_{esc} — the three values of N_{esc} are those with $T_{mrg} < 3$ Gyr, 1 Gyr and 100 Myr respectively.

Model name	$N(0)$	N_{sim}	$r_h(0)$ or R_s (pc)	$N_{BH}(0)$	N_{mrg}	t_{mrg} (Myr)	N_{esc}
Isolated clusters							
C10K20	10000	10	1.0	20	0	— —	— —
C25K50	25000	10	1.0	50	0	— —	3 1 1
C50K80	45000	1	1.0	80	1	698.3	3 1 0
C50K80.1	45000	1	0.5	80	2	217.1, 236.6	3 2 1
C50K40.1	45000	1	0.5	40	0	— —	1 1 1
C50K200	50000	1	1.0	200	2	100.8, 467.8	0 0 0
C65K110	65000	1	1.0	110	1	314.6	4 2 1
C65K110.1	65000	1	0.5	110	0	— —	4 3 1
C65K55.1	65000	1	0.5	55	1	160.5	1 0 0
C100K80	100000	1	1.0	80	2	219.4, 603.2	5 2 1
C100K200	100000	1	1.0	200	0	— —	5 4 4
Reflective boundary							
R3K180	3000	1	0.4	180	1	1723.9	5 3 1
R4K180A	4000	1	0.4	180	1	3008.8	2 2 1
R4K180B	4000	1	0.4	180	2	100.2, 1966.5	2 1 0
R3K100	3000	1	0.4	100	2	3052.8, 3645.9	1 1 0
R4K100A	4000	1	0.4	100	2	104.4, 814.2	3 3 1
R4K100B	4000	1	0.4	100	1	1135.3	3 3 3

massive cluster, where the BHs are concentrated after mass segregation. The advantage of this approach is that one can compute the evolution of a massive cluster with many fewer stars, allowing much faster computation. We integrate $N = 3000 - 4000$ stars bounded within 0.4 pc which gives a stellar density of $\sim 10^4 M_\odot \text{ pc}^{-3}$ appropriate for the core-density of a massive cluster. However, stars and BHs faster than a pre-assigned speed $v_{esc} \approx 24 \text{ km sec}^{-1}$, representing the escape speed from the parent cluster, are allowed to escape through the reflective boundary.

3. Results

Table 1 summarizes the results of our computations. For demonstration purposes, we choose one of the models, *viz.*, C50K80 (see Table 1) — other models generally possess similar properties. Fig. 1 (left panel) demonstrates the mass segregation of the BHs in the cluster which takes about 50 Myr. As the BHs segregate within about 0.3 pc of the cluster core, the BH density of becomes large enough to form BH-BH binaries through 3-body encounters. Once BH-BH binaries begin forming, single BHs and BH binaries start escaping from the BH-core due to the encounter-recoils (see Sec. 1). In Fig. 1 (left panel), one can clearly distinguish the two phases of the BH subsystem — the initial segregation phase and the BH-core formation, the radial positions of the BHs being scattered outwards in the latter phase due to the recoils. The corresponding decrease of N_{BH} during this phase is also shown in Fig. 1 (right panel).

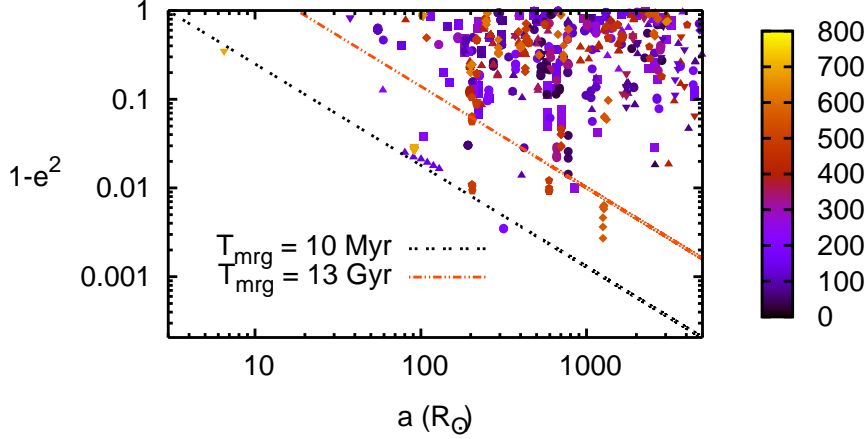


Figure 2: The positions of all the BH binaries within the cluster on the $1 - e^2$ vs. a plane where the different symbols are used to distinguish between different BH pairs (for model C50K80). The color-coding of the points, as indicated by the color-scale, represents the time (in Myr) at which they appear at a particular location in the above plane. The lines of constant GW merger times, $T_{\text{mrg}} = \text{constant}$, are overplotted.

3.1 Mergers and escapers

To study the possibility of BH-BH mergers, we consider the positions of the BH-BH binaries within the cluster in an a vs. $(1 - e^2)$ plane as shown in Fig. 2, where each pair is represented by a different symbol and are color-coded with the cluster evolution time in Myr. For each BH-pair, a and e fluctuate over the plane, the changes occurring over a collision time-scale which is $\sim \text{Myr}$. Since the orbital periods of the corresponding binaries are much shorter (from $\sim \text{days}$ to years), these points generally represent binaries which are stable over many orbits. Overplotted in Fig. 2 are the lines of constant GW merger time T_{mrg} as given by Eqn. (2.1). While most of the points correspond to very long merger times, a few of them do lie close to the $T_{\text{mrg}} = 10 \text{ Myr}$ line. This implies that these binaries are indeed hardened up to small enough a and/or acquire sufficient eccentricity that had they been left unperturbed, they would have merged via GW emission within several Myr. However, these merging BH-pairs can still be perturbed by further encounters on timescales $\sim \text{Myr}$ which can often prevent them from merging. In the particular example shown in Fig. 2, only one of them could merge within the cluster. This feature is found to be generally true for all the models reported here (see Table 1), *i.e.*, each of them does produce BH-pairs that are capable of merging within several Myr. On the other hand, among the escaped BH binaries, all of them with GW merger times smaller than a Hubble time are of interest since they remain unperturbed afterwards. For each of our computed models, a few of the escaped BH-BH binaries can merge in $T_{\text{mrg}} < 3 \text{ Gyr}$ as can be read from Table 1 (also, see reference [4]). We also find that for all of the computed models, most mergers occur within a few Gyr (see reference [4]).

The above results imply that an important class of star clusters for dynamically forming BH binaries that merge at the present epoch are those with initial mass $M_{\text{cl}}(0) \gtrsim 3 \times 10^4 M_{\odot}$ and are few Gyr old. Such clusters represent intermediate-age massive clusters (hereafter IMC) with initial masses close to the upper-limit of the initial cluster mass function (ICMF) in spiral and starburst galaxies [8]. Globular clusters, which are at least an order of magnitude more massive, are typi-

cally much older (~ 10 Gyr), so that they cannot contribute significantly to the present-day merger rate, as most of the mergers from them would have occurred earlier. On the other hand, young massive clusters, with ages < 50 Myr, are generally too young to produce BH-BH mergers, as the stratification of the BHs and the formation of the BH core itself takes longer. Hence, IMCs seem to be the most likely star clusters for dynamically generating present-day BH-BH merger events.

3.2 Detection rate of BH-BH mergers

The results from Table 1 give an average merger rate of $\approx 0.4/\text{cluster}/\text{Gyr}$ for those occurring within the clusters and $\approx 0.9/\text{cluster}/\text{Gyr}$ for the escapers, totaling $\mathcal{R}_{\text{mrg}} \approx 1.3/\text{cluster}/\text{Gyr}$ (see reference [4]). To estimate the detection rate of BH-BH mergers from IMCs, we adopt the space density, as derived by reference [9], for young populous clusters which is $\rho_{cl} \approx 3.5 h^3 \text{ Mpc}^{-3}$, where $h = 0.73$ is the Hubble parameter. This merger rate implies that the IMCs would yield a BH-BH merger detection rate of $\mathcal{R}_{\text{AdLIGO}} \approx 30 \text{ yr}^{-1}$ for the upcoming “Advanced LIGO” (AdLIGO) GW detector (see reference [4] and citations therein). Interestingly, the above dynamical BH-BH merger detection rate can be more than 10 times higher than that from individual primordial binaries [10]. Our results then imply that the dynamically formed BH-BH binaries constitute the dominant contribution to the BH-BH merger detection from the Universe [4].

4. Introduction of stellar evolution

Although the study described above has provided significant insights into the dynamics of stellar mass BHs in star clusters and its role in dynamically generating BH-BH merger events and has been performed self-consistently using direct N-body computations, the cluster models would be more realistic if the BHs are formed with a mass function from stellar evolution. To materialize such improvements, we model the initial Plummer clusters with all stars at their Zero-Age-Main-Sequence (ZAMS) having a Kroupa IMF [11] (upto $100M_{\odot}$). These models are also evolved using the NBODY6 integrator where the direct N-body integration of the cluster stars is coupled with their stellar evolution using analytic but well tested stellar evolution recipes [12].

Fig. 3 shows the evolution of the core radii r_c for two cluster models with different metallicities, viz., $Z = Z_{\odot}$ and $Z = 0.01Z_{\odot}$, having otherwise exactly the same initial conditions ($N(0) = 10^5$, $r_h(0) = 1 \text{ pc}$ Plummer spheres and all stars having identical positions, velocities and masses following a Kroupa IMF). All the BHs are retained in the clusters following their formation via supernovae. For the cluster with lower Z , the BH masses are considerably larger than that for the higher Z case (average BH mass $\approx 14M_{\odot}$ for $Z = 0.01Z_{\odot}$ as opposed to $\approx 7M_{\odot}$ for $Z = Z_{\odot}$) so that the expansion of the core radius due to the heating effect of the BH sub-cluster (see Sec. 1) is significantly more prominent for the lower Z model, as can be seen in Fig. 3. Also, the $Z = 0.01Z_{\odot}$ model have produced 3 BH-BH binaries among its escapers that can merge via GW radiation within a Hubble time while its high Z counterpart have produced only one. This can be expected as the heavier BHs in the lower Z cluster create a denser BH core where the binary BH formation rate through 3-body encounters and their subsequent dynamical hardening rates are higher. This indicates that the modification of the dynamical evolution of a cluster by its BHs and the dynamical BH-BH merger rate depend crucially on its metallicity. Studying the effect of the stellar content of a cluster and of its properties on the dynamics of its BHs is under progress.

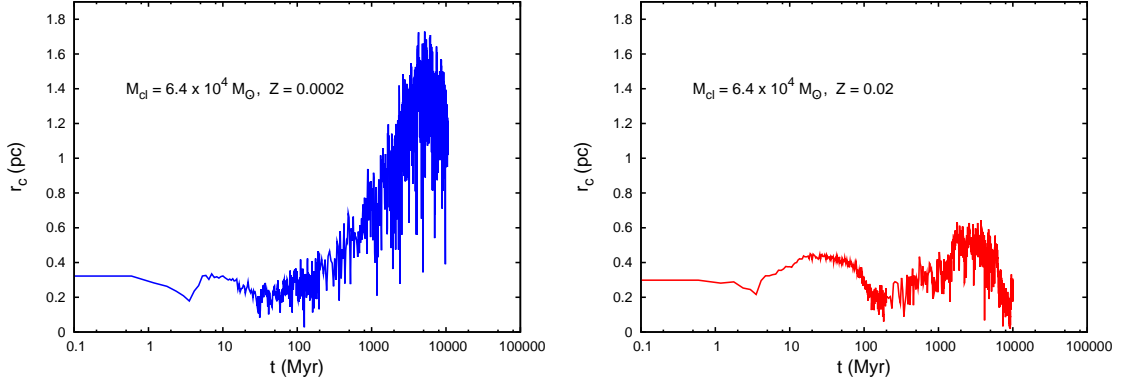


Figure 3: Evolution of core radii r_c for two computed clusters with metallicities $Z = Z_\odot$ and $Z = 0.01Z_\odot$ but otherwise identical initial conditions (see text). A full BH retention is assumed. Such comparison demonstrates that the heating effect of the BH core and the resulting expansion of r_c is significantly larger for the lower metallicity cluster (left panel) as the BHs formed are considerably more massive in that case.

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